

Version 8.5



# **Table of Contents**

IMSL® C NUMERICAL LIBRARY VERSION 8.5	2
Mathematical Functionality Overview	5
Statistical Functionality Overview	6
${ m IMSL}^{ m \tiny B}$ Libraries are also available for Fortran, Java, C# for .NET and Python	7
IMSL C MATH LIBRARY	8
CHAPTER 1: LINEAR SYSTEMS	Я
CHAPTER 2: EIGENSYSTEM ANALYSIS	
CHAPTER 3: INTERPOLATION AND APPROXIMATION	
CHAPTER 4: QUADRATURE	
CHAPTER 5: DIFFERENTIAL EQUATIONS	14
CHAPTER 6: TRANSFORMS	15
CHAPTER 7: NONLINEAR EQUATIONS	
CHAPTER 8: OPTIMIZATION	
CHAPTER 9: SPECIAL FUNCTIONS	
CHAPTER 10: STATISTICS AND RANDOM NUMBER GENERATION	
CHAPTER 11: PRINTING FUNCTIONS	-
CHAPTER 12: UTILITIES	28
IMSL C STAT LIBRARY	34
CHAPTER 1: BASIC STATISTICS	34
CHAPTER 2: REGRESSION	
CHAPTER 3: CORRELATION AND COVARIANCE	
CHAPTER 4: ANALYSIS OF VARIANCE AND DESIGNED EXPERIMENTS	
CHAPTER 5: CATEGORICAL AND DISCRETE DATA ANALYSIS	
CHAPTER 6: NONPARAMETRIC STATISTICS	
CHAPTER 7: TESTS OF GOODNESS OF FIT	
CHAPTER 8: TIME SERIES ANALYSIS AND FORECASTING	
CHAPTER 9: MULTIVARIATE ANALYSIS	
CHAPTER 10: SURVIVAL AND RELIABILITY ANALYSIS	
CHAPTER 11: PROBABILITY DISTRIBUTION FUNCTIONS AND INVERSES	
CHAPTER 12: RANDOM NUMBER GENERATION	
CHAPTER 13: DATA MINING	
CHAPTER 14: PRINTING FUNCTIONS	
CHAPTER 15: UTILITIES	56

# IMSL® C NUMERICAL LIBRARY VERSION 8.5

Advanced Mathematical and Statistical Numerical Library for C, C++, and Python Programmers

At the heart of the IMSL C Numerical Library is a comprehensive set of pre-built mathematical and statistical analysis functions that developers can embed directly into their applications. Available for a wide range of computing platforms, the robust, scalable, portable, and high performing IMSL analytics allow developers to focus on their domain of expertise and reduce development time.

#### **COST-EFFECTIVENESS AND VALUE**

The IMSL C Numerical Library significantly shortens application time to market and promotes standardization. Descriptive function names and variable argument lists have been implemented to simplify calling sequences. Using the IMSL C Library reduces costs associated with the design, development, documentation, testing, and maintenance of applications. The use of analytics that is robust, scalable, portable, and high performing inherently improves application reliability.

# A RICH SET OF DATA MINING FUNCTIONS

The IMSL C Library includes a comprehensive set of functions for data mining, modeling, prediction, and optimization. There are time series models such as ARIMA, GARCH, and vector auto-regression, plus decision trees, Apriori analysis, SVM, neural networks, linear and logistic regression, K-means clustering, Bayes classification, and much more. The IMSL C Library also includes functions for analyzing streaming data and working with big or distributed data.

#### **EMBEDDABILITY**

Development is made easier because library code readily embeds into application code, with no additional infrastructure such as app/management consoles, servers, or programming environments needed.

Wrappers complicate development by requiring the developer to access external compilers and pass arrays or user-defined data types to ensure compatibility between the different languages. The IMSL C Library allows developers to write, build, compile, and debug code in a single environment, and to easily embed analytic functions in applications and databases.

#### RELIABILITY

100 percent pure C code maximizes robustness. It allows faster and easier debugging, through superior error handling that not only conveys the error condition, but also suggests corrective action if appropriate. The result is reduced application support cost due to the minimization of user error and data issues.

The IMSL C Library has been rigorously tested by Rogue Wave, and seasoned by all industry verticals for over 40 years. You can expect consistent results across all supported platforms and languages, which makes platform migration and upgrade easy and efficient.

#### **HIGH PERFORMANCE**

The IMSL C Library integrates the CUDA BLAS library on systems with NVIDIA GPU hardware. For many linear algebra functions, work can be offloaded to the GPU for significantly enhanced performance. The calling sequences for the IMSL functions are unchanged, so there is no learning curve and users can be productive immediately.

The library is also designed to take advantage of symmetric multiprocessor (SMP) systems, both multi-CPU and multicore. Many algorithms leverage OpenMP directives on supported environments to distribute calculations across available resources. In areas such as linear algebra and Fast Fourier Transforms, third-party high-performance vendor libraries leverage SMP capabilities on a variety of systems.

#### **SCALABILITY**

The IMSL C Library supports scalability through:

- An enhanced ability to analyze time-sequenced data, or streaming real-time data that isn't stored.
- Improved algorithms that can analyze data sets too large to fit into memory or that exist on separate nodes.
- Memory management that ensures applications will not crash when they encounter a low memory condition.

### **USER FRIENDLY NOMENCLATURE**

The IMSL C Library uses descriptive, explanatory function names for intuitive programming that:

- Are easy to identify and use, and prevent conflicts with other software.
- Provide a common root name for numerical functions that offer the choice of multiple precisions.

# PROGRAMMING INTERFACE FLEXIBILITY

The IMSL C Library takes full advantage of the intrinsic characteristics and desirable features of the C language. The functions support variable-length argument lists, where the concise set of required arguments contains only information necessary for usage. Optional arguments provide added functionality and power to each function. Memory allocation can be handled by the library or by the developer. Finally, user-defined functions are implemented with interfaces that C developers will find natural.

#### **THREAD SAFETY**

The IMSL C Library is thread safe. Thread safety allows the IMSL C Library to be used in multi-threaded applications where performance benefits can be realized through concurrent and/or parallel execution.

### **COMPREHENSIVE DOCUMENTATION**

Documentation for the IMSL C Library is comprehensive, clearly written, and standardized. The documentation, in multiple formats:

- · Provides organized, easy-to-find information.
- Documents, explains, and provides references for algorithms.
- Gives at least one example of each function's usage, with sample input and result.

#### **UNMATCHED PRODUCT SUPPORT**

Behind every Rogue Wave license is a team of professionals ready to provide expert answers to questions about the IMSL Numerical Libraries.

#### Product support:

- Gives users direct access to Rogue Wave's resident staff of expert product support specialists.
- Provides prompt, two-way communication with solutions to a user's programming needs.
- · Includes product maintenance updates.

#### **CONSULTING SERVICES**

Rogue Wave offers expert consulting services for algorithm development as well as complete application development. Please contact Rogue Wave to learn more about its extensive experience in developing custom algorithms, building algorithms on scalable platforms, and full applications development.

### **ACCESS FROM PYTHON**

The IMSL C Library functions are also available within Python for rapid prototyping, using PyIMSL™, a collection of Python wrappers to the algorithms in the IMSL C Library. By using the same IMSL algorithms in the prototype as in the production code, developers can deliver accurate and consistent results in the production application sooner rather than later. For more information, please visit

http://www.roguewave.com/products/imsl-numerical-libraries/pyimsl-studio.aspx.

# **Mathematical Functionality Overview**

The IMSL C Numerical Library is a collection of the most commonly required numerical functions, tailored for a C programmer's needs. The mathematical functionality is organized into 12 sections. These capabilities range from solving systems of linear equations to optimization.

- Linear Systems including real and complex, full and sparse matrices, linear least squares, matrix decompositions, generalized inverses and vectormatrix operations. The least squares solvers may include non-negative and general linear constraints. Matrix decompositions now include non-negative, low-rank factorizations.
- Eigensystems Analysis including eigenvalues and eigenvectors of complex, real symmetric, and complex Hermitian matrices.
- Interpolation and Approximation including constrained curve-fitting splines, cubic splines, leastsquares approximation and smoothing, and scattered data interpolation.
- Integration and Differentiation including univariate, multivariate, Gauss quadrature, and quasi-Monte Carlo.
- Differential Equations using Adams-Gear and Runge-Kutta methods for stiff and non-stiff ordinary differential equations, with support for partial differential equations, including the Feynman-Kac solver. Also included are second-order ODE solvers, constrained DAE solvers, and a method of lines PDE solver.

- Transforms including real and complex, one- and two-dimensional Fast Fourier Transforms, as well as convolutions, correlations, and Laplace transforms.
- Nonlinear Equations including zeros and root finding of polynomials, zeros of a function and root of a system of equations.
- Optimization including unconstrained linearly and nonlinearly constrained minimizations and linear and quadratic programming interior point algorithms.
- Special Functions including error and gamma functions, real order complex valued Bessel functions and statistical functions.
- **Financial Functions** including functions for Bond and cash-flow analysis.
- Random Number Generation including a
  generator for multivariate normal distributions and
  pseudorandom numbers from several distributions,
  including gamma, Poisson, and beta. Also, support for
  low-discrepancy series using a generalized Faure
  sequence.
- Utilities including CPU time used, machine, mathematical, physical constants, retrieval of machine constants, and customizable error-handling.

# **Statistical Functionality Overview**

The statistical functionality is organized into 13 sections. These capabilities range from analysis of variance to random number generation.

- Basic Statistics including univariate summary statistics, frequency tables, and rank and order statistics.
- Regression including linear (multivariate),
  polynomial, and nonlinear regression models as well as
  robust alternatives such as Lpnorm and Partial Least
  Squares. This section also contains stepwise and all
  best-variable selection methods.
- Correlation and Covariance including sample variance-covariance, partial correlation and covariances, pooled variance-covariance, and robust estimates of a covariance matrix and mean factor.
- Analysis of Variance and Designed Experiments

   analysis of standard factorial experiments, randomized complete block designs, Latin-square, lattice, split-plot and strip-plot, and related experiments; analysis of hierarchical data, computation of false discovery rates, and standard tests for multiple comparisons and homogeneity of variance.
- Categorical and Discrete Data Analysis including chi-squared analysis of a two-way contingency
   table, exact probabilities in a two-way contingency table,
   logistic regression for binomial or multinomial responses,
   and the analysis of categorical data using general linear
   models.
- Nonparametric Statistics including sign tests, Wilcoxon sum tests, and Cochran Q test for related observations.
- Tests of Goodness-of-Fit including the chisquared goodness-of-fit test, the Kolmogorov/Smirnov one- and two-sample tests for continuous distributions, the Shapiro-Wilk, Lilliefors, chi-squared, Anderson-Darling, and Cramer-Von Mises tests for normality, Mardia's test for multivariate normality, and the runs, pairs-serial, σ², and triplets tests for randomness.

- Time Series Analysis and Forecasting analysis and forecasting of time series using a nonseasonal ARMA model, ARIMA with regression, Holt-Winters exponential smoothing, GARCH, Kalman filtering; various fitting and diagnostic utilities including portmanteau lack of fit test and difference of a seasonal or nonseasonal time series.
- Multivariate Analysis including principal component analysis, discriminant analysis, K-means and hierarchical cluster analysis, and factor analysis.
   Methods of factor analysis include principal factor, image analysis, unweighted least squares, generalized least squares, maximum likelihood, and various factor rotations.
- Survival Analysis including analysis of data using the Cox linear survival model, Kaplan-Meier survival estimates, actuarial survival tables, and non-parametric survival estimates.
- Probability Distribution Functions and Inverses including the cumulative distribution function (CDF), inverse CDF, and probability density function (PDF) for many common discrete and continuous distributions, as well as multivariate normal, non-central *F*, Chi-square, Beta, Students *t*, and others. This section also includes parameter estimation by maximum likelihood.
- Random Number Generation including generators for many univariate discrete and continuous distributions, as well as multivariate Normal, multinomial, Gaussian or Student's t copula, an ARMA or nonhomogeneous Poisson process, order statistics, permutations, and more. This section also allows a choice of pseudorandom number generators, including the Mersenne Twister.
- Data Mining including decision trees, vector autoregression, Apriori analysis, cluster analysis, Kohonen self-organizing maps, support vector machine, ensemble models, genetic algorithms, PCA, factor analysis, feed-forward neural networks, neural network data pre- and post-processing algorithms, and much more.

# IMSL® Numerical Libraries are also available for Fortran, Java, C# for .NET, and Python

# **IMSL®** Fortran Numerical Library

The IMSL® Fortran Numerical Library is the gold standard mathematical and statistical code library for Fortran programmers developing high-performance computing applications. The IMSL Fortran Library contains highly accurate and reliable Fortran algorithms with full coverage of mathematics and statistics and complete backward compatibility.

The IMSL Fortran Library is a comprehensive library of mathematical and statistical algorithms available in one cohesive package. It combines the powerful and flexible interface features of the Fortran language with the performance benefits of both distributed memory and shared memory multiprocessing architectures.

# JMSL™ Numerical Library for Java Applications

The JMSL Numerical Library for Java applications is the broadest collection of mathematical, statistical, financial, data mining and charting classes available in 100 percent Java. It is the only Java programming solution that combines integrated charting with the reliable mathematical and statistical functionality of the industry-leading IMSL Numerical Library algorithms. This blend of advanced numerical analysis and visualization on the Java platform allows organizations to gain insight into valuable data and share analysis results across the enterprise quickly. The JMSL Library continues to be the leader, providing robust and portable data analysis and visualization technology for the Java platform, and a fast, scalable framework for tailored analytical applications.

# IMSL C# Numerical Library for Microsoft<sup>®</sup>.NET Applications

The IMSL C# Numerical Library for Microsoft® .NET
Applications is a numerical analysis library written in 100
percent C#, providing broad coverage of advanced
mathematics and statistics for the .NET Framework. This
offers .NET developers seamless accessibility to analytics
capabilities in the most integrated language for the .NET
environment with the highest degree of programming
productivity and ease of use with Visual Studio®.

The IMSL C# Library is the only numerical library to offer industry standard numerical analysis and charting for .NET languages. This Library provides unprecedented analytic capabilities and the most comprehensive and accessible mathematical, statistical, and finance algorithms for.NET languages. With the IMSL C# Library, IMSL has brought all of the benefits inherent in the .NET Framework to a new level by adding robust analytics to its broad set of capabilities. Because the C# Library is written in C#, it is easily integrated into any .NET language, such as Visual Basic .NET, F#, and IronPython among others.

### **PyIMSL™ Studio**

PyIMSL Studio is the only commercially-available numerical analysis application development environment designed for deploying mathematical and statistical prototype models into production applications. PyIMSL Studio closes the prototype to production gap by providing modelers and implementation teams with a common set of tested and supported high-quality development tools, as well as the same underlying numerical algorithms. Using PyIMSL Studio, prototype work is transformed into production applications faster, with less complexity, cost, and risk to the project.

# **IMSL C MATH LIBRARY**

### **CHAPTER 1: LINEAR SYSTEMS**

LINEAR EQUATIONS WITH FULL MAT	LINEAR EQUATIONS WITH FULL MATRICES		
FUNCTION	DESCRIPTION		
lin_sol_gen	Solves a real general system of linear equations $Ax = b$ .		
lin_sol_gen (complex)	Solves a complex general system of linear equations $Ax = b$ .		
lin_sol_posdef	Solves a real symmetric positive definite system of linear equations $Ax = b$ .		
lin_sol_posdef (complex)	Solves a complex Hermitian positive definite system of linear equations $Ax = b$ .		
LINEAR EQUATIONS WITH BAND MATRICES			
FUNCTION	DESCRIPTION		
lin_sol_gen_band	Solves a real general band system of linear equations $Ax = b$ .		
lin_sol_gen_band (complex)	Solves a complex general band system of linear equations $Ax = b$ .		
lin_sol_posdef_band	Solves a real symmetric positive definite system of linear equations $Ax = b$ in band symmetric storage mode.		
lin_sol_posdef_band (complex)	Solves a complex Hermitian positive definite system of linear equations $Ax = b$ in band symmetric storage mode.		
LINEAR EQUATIONS WITH GENERAL	L SPARSE MATRICES		
FUNCTION	DESCRIPTION		
lin_sol_gen_coordinate	Solves a sparse system of linear equations $Ax = b$ .		
lin_sol_gen_coordinate (complex)	Solves a system of linear equations $Ax = b$ , with sparse complex coefficient matrix $A$ .		
superlu	Computes the $LU$ factorization of a general sparse matrix by a column method and solves the real sparse linear system of equations $Ax = b$ .		

FUNCTION	DESCRIPTION
FUNCTION	
superlu (complex)	Computes the $LU$ factorization of a general complex sparse matrix by a column method and solves the complex sparse linear system of equations $Ax = b$ .
superlu_smp	Computes the $LU$ factorization of a general sparse matrix by a left looking column method using OpenMP parallelism, and solves the real sparse linear system of equations $Ax = b$ .
superlu_smp (complex)	Computes the $LU$ factorization of a general complex sparse matrix by a left-looking column method using OpenMP parallelism and solves the complex sparse linear system of equations $Ax = b$ .
lin_sol_posdef_coordinate	Solves a sparse real symmetric positive definite system of linear equations $Ax = b$ .
lin_sol_gen_posdef_coordinate (comp	Solves a sparse Hermitian positive definite system of linear equations $Ax = b$ .
sparse_cholesky_smp	Computes the Cholesky factorization of a sparse real symmetric positive definite matrix $A$ by an OpenMP parallelized supernodal algorithm and solves the sparse real positive definite system of linear equations $Ax = b$ .
sparse_cholesky_smp (complex)	Computes the Cholesky factorization of a sparse (complex) Hermitian positive definite matrix $A$ by an OpenMP parallelized supernodal algorithm and solves the sparse Hermitian positive definite system of linear equations $Ax = b$ .
lin_sol_gen_min_residual	Solves a linear system $Ax = b$ using the restarted generalized minimum residual (GMRES) method.
lin_sol_def_cg	Solves a real symmetric definite linear system using a conjugate gradient method.
ITERATIVE METHODS	
FUNCTION	DESCRIPTION
lin_sol_gen_min_residual	Solves a linear system $Ax = b$ using the restarted generalized minimum residual (GMRES) method.
lin_sol_def_cg	Solves a real symmetric definite linear system using a conjugate gradient method.
LINEAR LEAST-SQUARES WITH FULL M	ATRICES
FUNCTION	DESCRIPTION

LINEAR LEAST-SQUARES WITH	LINEAR LEAST-SQUARES WITH FULL MATRICES	
FUNCTION	DESCRIPTION	
lin_least_squares_gen	Solves a linear least-squares problem $Ax = b$ .	
nonneg_least_squares	Computes the non-negative least squares (NNLS) solution of an $m \times n$ real linear least squares system, $Ax \cong b, x \geq 0$ .	
lin_lsq_lin_constraints	Solves a linear least-squares problem with linear constraints.	
nonneg_matrix_factorization	Given an $m \times n$ real matrix $A \ge 0$ and an integer $k \le \min(m, n)$ , computes a factorization $A \cong FG$ .	
lin_svd_gen	Computes the SVD, $A = USV^T$ , of a real rectangular matrix $A$ .	
lin_svd_gen (complex)	Computes the SVD, $A = USV^T$ , of a complex rectangular matrix $A$ .	
lin_sol_nonnegdef	Solves a real symmetric nonnegative definite system of linear equations $Ax = b$ .	

# **CHAPTER 2: EIGENSYSTEM ANALYSIS**

LINEAR EIGENSYSTEM PROBLEMS	LINEAR EIGENSYSTEM PROBLEMS	
FUNCTION	DESCRIPTION	
eig_gen	Computes the eigenexpansion of a real matrix $A$ .	
eig_gen (complex)	Computes the eigenexpansion of a complex matrix. A.	
eig_sym	Computes the eigenexpansion of a real symmetric matrix A.	
eig_herm (complex)	Computes the eigenexpansion of a complex Hermitian matrix $A$ .	
GENERALIZED EIGENSYSTEM PROBLEM	GENERALIZED EIGENSYSTEM PROBLEMS	
FUNCTION	DESCRIPTION	
eig_symgen	Computes the generalized eigenexpansion of a system $Ax = \lambda Bx$ . $A$ and $B$ are real and symmetric. $B$ is positive definite.	

FUNCTION	DESCRIPTION
geneig	Computes the generalized eigenexpansion of a system $Ax = \lambda Bx$ , with $A$ and $B$ real.
geneig (complex)	Computes the generalized eigenexpansion of a system $Ax = \lambda Bx$ , with $A$ and $B$ complex.

# **CHAPTER 3: INTERPOLATION AND APPROXIMATION**

CUBIC SPLINE INTERPOLATION		
FUNCTION	DESCRIPTION	
cub_spline_interp_e_cnd	Computes a cubic spline interpolant, specifying various endpoint conditions.	
 cub_spline_interp_shape	Computes a shape-preserving cubic spline.	
 cub_spline_tcb	Computes a tension-continuity-bias (TCB) cubic spline interpolant. This is also called a Kochanek-Bartels spline and is a generalization of the Catmull-Rom spline.	
CUBIC SPLINE EVALUATION AND INTEG	GRATION	
FUNCTION	DESCRIPTION	
cub_spline_value	Computes the value of a cubic spline or the value of one of its derivatives.	
cub_spline_integral	Computes the integral of a cubic spline.	
SPLINE INTERPOLATION		
FUNCTION	DESCRIPTION	
spline_interp	Computes a spline interpolant.	
spline_knots	Computes the knots for a spline interpolant.	
spline_2d_interp	Computes a two-dimensional, tensor-product spline interpolant from two-dimensional, tensor-product data.	
SPLINE EVALUATION AND INTEGRATIO		
 FUNCTION	DESCRIPTION	
spline_value	Computes the value of a spline or the value of one of its derivatives.	

SPLINE EVALUATION AND INTEGRATION	ON
FUNCTION	DESCRIPTION
spline_integral	Computes the integral of a spline.
spline_2d_value	Computes the value of a tensor-product spline or the value of one of its partial derivatives.
 spline_2d_integral	Evaluates the integral of a tensor-product spline on a rectangular domain.
MULTIDIMENSIONAL INTERPOLATION	
FUNCTION	DESCRIPTION
spline_nd_interp	Performs multidimensional interpolation and differentiation for up to 7 dimensions.
LEAST-SQUARES APPROXIMATION AND SMOOTHING	
FUNCTION	DESCRIPTION
user_fcn_least_squares	Computes a least-squares fit using user-supplied functions.
spline_least_squares	Computes a least-squares spline approximation.
spline_2d_least_squares	Computes a two-dimensional, tensor-product spline approximant using least squares.
cub_spline_smooth	Computes a smooth cubic spline approximation to noisy data by using cross-validation to estimate the smoothing parameter or by directly choosing the smoothing parameter.
spline_lsq_constrained	Computes a least-squares constrained spline approximation.
smooth_1d_data	Smooth one-dimensional data by error detection.
SCATTERED DATA INTERPOLATION	
FUNCTION	DESCRIPTION
scattered_2d_interp	Computes a smooth bivariate interpolant to scattered data that is locally a quintic polynomial in two variables.

SCATTERED DATA LEAST-SQUARES	
FUNCTION	DESCRIPTION
radial_scattered_fit	Computes an approximation to scattered data in $\mathbb{R}^n$ for $n \ge 1$ using radial-basis functions.
radial_evaluate	Evaluates a radial basis fit.

# **CHAPTER 4: QUADRATURE**

UNIVARIATE QUADRATURE	
FUNCTION	DESCRIPTION
int_fcn_sing	Integrates a function, which may have endpoint singularities, using a globally adaptive scheme based on Gauss-Kronrod rules.
int_fcn_sing_1d	Integrates a function with a possible internal or endpoint singularity.
int_fcn	Integrates a function using a globally adaptive scheme based on Gauss-Kronrod rules.
int_fcn_sing_pts	Integrates a function with singularity points given.
int_fcn_alg_log	Integrates a function with algebraic-logarithmic singularities.
int_fcn_inf	Integrates a function over an infinite or semi-infinite interval.
int_fcn_trig	Integrates a function containing a sine or a cosine factor.
int_fcn_fourier	Computes a Fourier sine or cosine transform.
int_fcn_cauchy	Computes integrals of the form $\int_a^b \frac{f\left(x\right)}{x-c}  dx$ in the Cauchy principal value sense.
int_fcn_smooth	Integrates a smooth function using a nonadaptive rule.

MULTIDIMENSIONAL QUADRATURE FUNCTION	DESCRIPTION	
int_fcn_2d	Computes a two-dimensional iterated integral.	
int_fcn_sing_2d	Integrates a function of two variables with a possible internal or endpoint singularity.	
int_fcn_sing_3d	Integrates a function of three variables with a possible internal or endpoint singularity.	
int_fcn_hyper_rect	Integrate a function on a hyper-rectangle, $\int_{a_0}^{b_0} \dots \int_{a_{n-1}}^{b_{n-1}} f\left(x_0,\dots,x_{n-1}\right) dx_{n-1}\dots dx_0$	
int_fcn_qmc	Integrates a function over a hyper-rectangle using a quasi-Monte Carlo method.	
GAUSS RULES AND THREE-TERM RECURRENCES		
FUNCTION	DESCRIPTION	
gauss_quad_rule	Computes a Gauss, Gauss-Radau, or Gauss-Lobatto quadrature rule with various classical weight functions.	
DIFFERENTIATION		
FUNCTION	DESCRIPTION	
fcn_derivative	Computes the first, second, or third derivative of a user-supplied function.	

# **CHAPTER 5: DIFFERENTIAL EQUATIONS**

	FIRST-ORDER ORDINARY DIFFERENTIAL EQUATIONS		
	SOLUTION OF THE INITIAL VALUE PROBLEM	SOLUTION OF THE INITIAL VALUE PROBLEM FOR ODES	
	FUNCTION	DESCRIPTION	
	ode_runge_kutta	Solves an initial-value problem for ordinary differential equations using the Runge-Kutta-Verner fifth-order and sixth-order method.	
	SOLUTION OF THE BOUNDARY VALUE PROBLEM FOR ODES		
_	FUNCTION	DESCRIPTION	
	bvp_finite_difference	Solves a (parameterized) system of differential equations with boundary conditions at two points, using a variable order, variable step size finite difference method with deferred corrections.	

SOLUTION OF DIFFERENTIAL-ALGEB	RAIC SYSTEMS
FUNCTION	DESCRIPTION
differential_algebraic_eqs	Solves a first order differential-algebraic system of equations, $g(t, y, y') = 0$ , with optional additional constraints and user-defined linear system solver.
FIRST-AND-SECOND ORDER ORD	INARY DIFFERENTIAL EQUATIONS
SOLUTION OF THE INITIAL VALUE PR	OBLEM FOR ODES
FUNCTION	DESCRIPTION
ode_adams_krogh	Solves an initial-value problem for a system of ordinary differential equations of order one or two using a variable order Adams method.
PARTIAL DIFFERENTIAL EQUATION	DNS
SOLUTION OF SYSTEMS OF PDES IN	ONE DIMENSION
FUNCTION	DESCRIPTION
pde_1d_mg	Solves a system of one-dimensional time-dependent partial differential equations using a moving-grid interface.
modified_method_of_lines	Solves a system of partial differential equations of the form $u_t = f(x, t, u, u_x, u_{xx})$ using the method of lines.
feynman_kac	Solves a generalized Feynman-Kac equation on a finite interval using Hermite quintic splines.
feynman_kac_evaluate	Computes the value of a Hermite quintic spline or the value of one of its derivatives.
SOLUTION OF A PDE IN TWO DIMENS	SIONS
FUNCTION	DESCRIPTION
fast_poisson_2d	Solves Poisson's or Helmholtz's equation on a two-dimensional rectangle using a fast Poisson solver based on the HODIE finite-difference scheme on a uniform mesh.

# **CHAPTER 6: TRANSFORMS**

REAL TRIGONOMETRIC FFT	
FUNCTION	DESCRIPTION
fft_real	Computes the discrete Fourier transform of a real sequence.
fft_real_init	Computes the parameters for imsl_f_fft_real.

COMPLEX EXPONENTIAL FFT	
FUNCTION	DESCRIPTION
fft_complex	Computes the complex discrete Fourier transform of a complex sequence.
 fft_complex_init	Computes the parameters for imsl_c_fft_complex.
REAL SINE AND COSINE FFTS	
FUNCTION	DESCRIPTION
fft_cosine	Computes the discrete Fourier cosine transformation of an even sequence.
fft_cosine_init	Computes parameters needed by imsl_f_fft_cosine.
 fft_sine	Computes the discrete Fourier sine transformation of an odd sequence.
fft_sine_init	Computes parameters needed by imsl_f_fft_cosine.
TWO-DIMENSIONAL FFTS	
FUNCTION	DESCRIPTION
 fft_2d_complex	Computes the complex discrete two-dimensional Fourier transform of a complex two-dimensional array.
CONVOLUTIONS AND CORRELATIONS	
FUNCTION	DESCRIPTION
convolution	Computes the convolution of two real vectors.
convolution (complex)	Computes the convolution, and optionally, the correlation of two complex vectors.
LAPLACE TRANSFORM	
 FUNCTION	DESCRIPTION
 inverse_laplace	Computes the inverse Laplace transform of a complex function.

# **CHAPTER 7: NONLINEAR EQUATIONS**

ZEROS OF A POLYNOMIAL	
FUNCTION	DESCRIPTION
zeros_poly	Finds the zeros of a polynomial with real coefficients using the Jenkins-Traub three-stage algorithm.
zeros_poly (complex)	Finds the zeros of a polynomial with complex coefficients using the Jenkins-Traub three-stage algorithm.
ZEROS OF A FUNCTION	
FUNCTION	DESCRIPTION
zero_univariate	Finds a zero of a real univariate function.
zero_function	Finds the real zeros of a real, continuous, univariate function.
ROOT OF A SYSTEM OF EQUATIONS	
FUNCTION	DESCRIPTION
zeros_sys_eqn	Solves a system of n nonlinear equations $f(x) = 0$ using a modified Powell hybrid algorithm.

# **CHAPTER 8: OPTIMIZATION**

UNCONSTRAINED MINIMIZATION	UNCONSTRAINED MINIMIZATION	
UNIVARIATE FUNCTION		
FUNCTION	DESCRIPTION	
min_uncon	Finds the minimum point of a smooth function $f(x)$ of a single variable using only function evaluations.	
min_uncon_deriv	Finds the minimum point of a smooth function of a single variable using both function and first derivative evaluations.	
min_uncon_golden	Finds the minimum point of a nonsmooth function of a single variable using the golden selection search method.	
MULTIVARIATE FUNCTION		
FUNCTION	DESCRIPTION	
min_uncon_multivar	Minimizes a function $f(x)$ of n variables using a quasi-Newton method.	

NONLINEAR LEAST-SQUARES	
FUNCTION	DESCRIPTION
nonlin_least_squares	Solves a nonlinear least-squares problem using a modified Levenberg- Marquardt algorithm.
LINEARLY CONSTRAINED MINIMIZATION	
FUNCTION	DESCRIPTION
read_mps	Reads an MPS file containing a linear programming problem or a quadratic programming problem.
 linear_programming	Solves a linear programming problem.
 lin_prog	Solves a linear programming problem using the revised simplex algorithm.
 quadratic_prog	Solves a quadratic programming problem subject to linear equality or inequality constraints.
sparse_lin_prog	Solves a sparse linear programming problem by an infeasible primal-dual interior-point method.
sparse_quadratic_prog	Solves a sparse convex quadratic programming problem by an infeasible primal-dual interior-point method.
min_con_gen_lin	Minimizes a general objective function subject to linear equality/inequality constraints.
 bounded_least_squares	Solves a nonlinear least-squares problem subject to bounds on the variables using a modified Levenberg-Marquardt algorithm.
NONLINEARLY CONCERNING AND	
NONLINEARLY CONSTRAINED MINIMIZAT	
 FUNCTION	DESCRIPTION
constrained_nlp	Solves a general nonlinear programming problem using a sequential equality constrained quadratic programming method.
SERVICE ROUTINES	
FUNCTION	DESCRIPTION
jacobian	Approximates the Jacobian of $\it m$ functions in $\it n$ unknowns using divided differences.

# **CHAPTER 9: SPECIAL FUNCTIONS**

ERROR AND GAMMA FUNCTIONS	ERROR AND GAMMA FUNCTIONS	
ERROR FUNCTIONS	ERROR FUNCTIONS	
FUNCTION	DESCRIPTION	
erf	Evaluates the real error function erf(x).	
erfc	Evaluates the real complementary error function erfc(x).	
erf_inverse	Evaluates the real inverse error function erf $^{-1}(x)$ .	
erfce	Evaluates the exponentially scaled complementary error function.	
erfe	Evaluates a scaled function related to erfc(z).	
erfc_inverse	Evaluates the real inverse complementary error function erfc $^{-1}(x)$ .	
beta	Evaluates the real beta function $\beta(x, y)$ .	
log_beta	Evaluates the logarithm of the real beta function $\beta(x, y)$ .	
beta_incomplete	Evaluates the real incomplete beta function $I_x = \beta_x(a,b)/\beta(a,b)$ .	
GAMMA FUNCTIONS		
FUNCTION	DESCRIPTION	
gamma	Evaluates the real gamma function $\Gamma(x)$ .	
log_gamma	Evaluates the logarithm of the absolute value of the gamma function log $ \Gamma(x) $ .	
gamma_incomplete	Evaluates the incomplete gamma function $\gamma(a, x)$ .	

PSI FUNCTIONS  PSI  Evaluates the derivative of the log gamma function.  PSI  Evaluates the second derivative of the log gamma function.  BESSEL FUNCTIONS  FUNCTION  DESCRIPTION  DESCRIPT		
Evaluates the derivative of the log gamma function.  Evaluates the second derivative of the log gamma function.  BESSEL FUNCTIONS  FUNCTION  DESCRIPTION  DESCRIPTION  Evaluates the real Bessel function of the first kind of order zero J <sub>0</sub> (x).  Dessel_J1  Evaluates the real Bessel function of the first kind of order one J <sub>1</sub> (x).  Dessel_Jx  Evaluates a sequence of Bessel function of the first kind with real order and complex arguments.  Dessel_Y0  Evaluates the real Bessel function of the second kind of order zero Y <sub>0</sub> (x).  Dessel_Y1  Evaluates the real Bessel function of the second kind of order one Y <sub>1</sub> (x).  Dessel_Y1  Evaluates the real Bessel function of the second kind of order one Y <sub>1</sub> (x).  Dessel_Y2  Evaluates a sequence of Bessel functions of the second kind of order one Y <sub>1</sub> (x).  Dessel_Y2  Evaluates a sequence of Bessel function of the first kind of order zero J <sub>0</sub> (x).  Evaluates the real modified Bessel function of the first kind of order zero J <sub>0</sub> (x).  Evaluates the real modified Bessel function of the first kind of order zero J <sub>0</sub> (x).  Dessel_I1  Evaluates the real modified Bessel function of the first kind of order one J <sub>1</sub> (x).  Evaluates the real modified Bessel function of the first kind of order zero J <sub>0</sub> (x).  Dessel_I2  Evaluates a sequence of modified Bessel function of the first kind of order one J <sub>1</sub> (x).  Evaluates a sequence of modified Bessel function of the first kind of order order one zero J <sub>0</sub> (x).  Dessel_Ix  Evaluates a sequence of modified Bessel function of the first kind of order zero J <sub>0</sub> (x).  Evaluates the real modified Bessel function of the first kind of order zero J <sub>0</sub> (x).  Evaluates the real modified Bessel function of the first kind of order zero J <sub>0</sub> (x).	PSI FUNCTIONS	
PSIL EVALUATES the second derivative of the log gamma function.  BESSEL FUNCTIONS FUNCTION  DESCRIPTION  DESCRIPTION  Evaluates the real Bessel function of the first kind of order zero J <sub>0</sub> (x).  Dessel_J1  Evaluates the real Bessel function of the first kind of order one J <sub>1</sub> (x).  Dessel_JX  Evaluates a sequence of Bessel functions of the first kind with real order and complex arguments.  Dessel_Y0  Evaluates the real Bessel function of the second kind of order zero Y <sub>0</sub> (x).  Dessel_Y1  Evaluates the real Bessel function of the second kind of order one Y <sub>1</sub> (x).  Dessel_YX  Evaluates the real Bessel function of the second kind of order one Y <sub>1</sub> (x).  Dessel_YX  Evaluates a sequence of Bessel functions of the second kind with real order and complex arguments.  Dessel_O0  Evaluates the real modified Bessel function of the first kind of order zero I <sub>0</sub> (x).  Dessel_exp_I0  Evaluates the exponentially scaled modified Bessel function of the first kind of order zero.  Dessel_exp_I1  Evaluates the exponentially scaled modified Bessel function of the first kind of order one I <sub>1</sub> (x).  Dessel_Exp_I1  Evaluates the exponentially scaled modified Bessel function of the first kind of order one I <sub>1</sub> (x).  Dessel_K0  Evaluates the exponentially scaled modified Bessel function of the first kind with real order and complex arguments.	FUNCTION	DESCRIPTION
BESSEL FUNCTIONS FUNCTION  DESCRIPTION  DESCRIPTION  Evaluates the real Bessel function of the first kind of order zero J <sub>0</sub> (x).  Dessel_J1  Evaluates the real Bessel function of the first kind of order one J <sub>1</sub> (x).  Dessel_JX  Evaluates a sequence of Bessel functions of the first kind with real order and complex arguments.  Dessel_Y0  Evaluates the real Bessel function of the second kind of order zero Y <sub>0</sub> (x).  Dessel_Y1  Evaluates the real Bessel function of the second kind of order zero Y <sub>0</sub> (x).  Dessel_Yx  Evaluates a sequence of Bessel functions of the second kind with real order and complex arguments.  Dessel_Yx  Evaluates a sequence of Bessel functions of the second kind with real order and complex arguments.  Dessel_exp_10  Evaluates the real modified Bessel function of the first kind of order zero I <sub>0</sub> (x).  Dessel_exp_10  Evaluates the real modified Bessel function of the first kind of order one I <sub>1</sub> (x).  Dessel_exp_11  Evaluates the real modified Bessel function of the first kind of order one I <sub>1</sub> (x).  Dessel_kx  Evaluates the exponentially scaled modified Bessel function of the first kind of order one I <sub>1</sub> (x).  Dessel_kx  Evaluates the real modified Bessel functions of the first kind with real order and complex arguments.	psi	Evaluates the derivative of the log gamma function.
PUNCTION  DESCRIPTION  Evaluates the real Bessel function of the first kind of order zero J <sub>0</sub> (x).  Dessel_J1  Evaluates the real Bessel function of the first kind of order zero J <sub>0</sub> (x).  Dessel_Jx  Evaluates a sequence of Bessel functions of the first kind with real order and complex arguments.  Dessel_Y0  Evaluates the real Bessel function of the second kind of order zero Y <sub>0</sub> (x).  Dessel_Y1  Evaluates the real Bessel function of the second kind of order zero Y <sub>0</sub> (x).  Dessel_Yx  Evaluates the real Bessel functions of the second kind with real order and complex arguments.  Dessel_Yx  Evaluates a sequence of Bessel functions of the second kind with real order and complex arguments.  Dessel_10  Evaluates the real modified Bessel function of the first kind of order zero I <sub>0</sub> (x).  Dessel_exp_10  Evaluates the exponentially scaled modified Bessel function of the first kind of order zero.  Dessel_11  Evaluates the exponentially scaled modified Bessel function of the first kind of order one I <sub>1</sub> (x).  Dessel_exp_11  Evaluates the exponentially scaled modified Bessel function of the first kind of order one order and complex arguments.  Dessel_kx  Evaluates the real modified Bessel function of the first kind with real order and complex arguments.  Dessel_K0  Evaluates the real modified Bessel function of the second kind of order zero K <sub>0</sub> (x).	psi1	Evaluates the second derivative of the log gamma function.
PUNCTION  DESCRIPTION  Evaluates the real Bessel function of the first kind of order zero J <sub>0</sub> (x).  Dessel_J1  Evaluates the real Bessel function of the first kind of order zero J <sub>0</sub> (x).  Dessel_Jx  Evaluates a sequence of Bessel functions of the first kind with real order and complex arguments.  Dessel_Y0  Evaluates the real Bessel function of the second kind of order zero Y <sub>0</sub> (x).  Dessel_Y1  Evaluates the real Bessel function of the second kind of order zero Y <sub>0</sub> (x).  Dessel_Yx  Evaluates the real Bessel functions of the second kind with real order and complex arguments.  Dessel_Yx  Evaluates a sequence of Bessel functions of the second kind with real order and complex arguments.  Dessel_10  Evaluates the real modified Bessel function of the first kind of order zero I <sub>0</sub> (x).  Dessel_exp_10  Evaluates the exponentially scaled modified Bessel function of the first kind of order zero.  Dessel_11  Evaluates the exponentially scaled modified Bessel function of the first kind of order one I <sub>1</sub> (x).  Dessel_exp_11  Evaluates the exponentially scaled modified Bessel function of the first kind of order one order and complex arguments.  Dessel_kx  Evaluates the real modified Bessel function of the first kind with real order and complex arguments.  Dessel_K0  Evaluates the real modified Bessel function of the second kind of order zero K <sub>0</sub> (x).	DESCEI ELINCTIONS	
bessel_J0  Evaluates the real Bessel function of the first kind of order zero J <sub>0</sub> (x).  Evaluates the real Bessel function of the first kind of order one J <sub>1</sub> (x).  bessel_Jx  Evaluates a sequence of Bessel functions of the first kind with real order and complex arguments.  bessel_Y0  Evaluates the real Bessel function of the second kind of order zero Y <sub>0</sub> (x).  bessel_Y1  Evaluates the real Bessel function of the second kind of order zero Y <sub>0</sub> (x).  bessel_Y1  Evaluates a sequence of Bessel function of the second kind of order one Y <sub>1</sub> (x).  bessel_Y2  Evaluates a sequence of Bessel function of the second kind with real order and complex arguments.  bessel_I0  Evaluates a sequence of Bessel function of the first kind of order zero I <sub>0</sub> (x).  bessel_exp_I0  Evaluates the real modified Bessel function of the first kind of order zero I <sub>0</sub> (x).  bessel_exp_I1  Evaluates the exponentially scaled modified Bessel function of the first kind of order one I <sub>1</sub> (x).  bessel_exp_I1  Evaluates the real modified Bessel function of the first kind of order one I <sub>1</sub> (x).  bessel_k0  Evaluates the real modified Bessel function of the second kind of order zero I <sub>0</sub> (x).  Evaluates the real modified Bessel function of the first kind with real order and complex arguments.  bessel_K0  Evaluates the real modified Bessel function of the second kind of order zero I <sub>0</sub> (x).		DESCRIPTION
Evaluates a sequence of Bessel functions of the first kind with real order and complex arguments.  bessel_Y0  Evaluates the real Bessel function of the second kind of order zero Y <sub>0</sub> (x).  bessel_Y1  Evaluates the real Bessel function of the second kind of order one Y <sub>1</sub> (x).  bessel_Yx  Evaluates a sequence of Bessel functions of the second kind with real order and complex arguments.  bessel_10  Evaluates the real modified Bessel function of the first kind of order zero I <sub>0</sub> (x).  bessel_exp_10  Evaluates the exponentially scaled modified Bessel function of the first kind of order zero.  bessel_exp_11  Evaluates the real modified Bessel function of the first kind of order one I <sub>1</sub> (x).  bessel_exp_11  Evaluates the exponentially scaled modified Bessel function of the first kind of order one.  bessel_tx  Evaluates a sequence of modified Bessel functions of the first kind with real order and complex arguments.  bessel_K0  Evaluates the real modified Bessel function of the second kind of order zero I <sub>0</sub> (x).		
bessel_Y0  Evaluates the real Bessel function of the second kind of order zero Y <sub>0</sub> (x).  bessel_Y1  Evaluates the real Bessel function of the second kind of order one Y <sub>1</sub> (x).  bessel_Yx  Evaluates a sequence of Bessel functions of the second kind with real order and complex arguments.  bessel_I0  Evaluates the real modified Bessel function of the first kind of order zero I <sub>0</sub> (x).  bessel_exp_I0  Evaluates the exponentially scaled modified Bessel function of the first kind of order zero.  bessel_I1  Evaluates the real modified Bessel function of the first kind of order one I <sub>1</sub> (x).  bessel_exp_I1  Evaluates the exponentially scaled modified Bessel function of the first kind of order one.  Evaluates the exponentially scaled modified Bessel functions of the first kind with real order one.  Evaluates a sequence of modified Bessel functions of the first kind with real order and complex arguments.  Evaluates the real modified Bessel function of the second kind of order zero K <sub>0</sub> (x).  Evaluates the real modified Bessel function of the second kind of order zero K <sub>0</sub> (x).  Evaluates the real modified Bessel function of the second kind of order zero K <sub>0</sub> (x).	bessel_J1	Evaluates the real Bessel function of the first kind of order one $J_1(x)$ .
bessel_Yx  Evaluates the real Bessel function of the second kind of order one Y <sub>1</sub> (x).  bessel_Yx  Evaluates a sequence of Bessel functions of the second kind with real order and complex arguments.  bessel_lo  Evaluates the real modified Bessel function of the first kind of order zero I <sub>0</sub> (x).  bessel_exp_lo  Evaluates the exponentially scaled modified Bessel function of the first kind of order zero.  bessel_l1  Evaluates the real modified Bessel function of the first kind of order one I <sub>1</sub> (x).  bessel_exp_l1  Evaluates the exponentially scaled modified Bessel function of the first kind of order one.  Evaluates the exponentially scaled modified Bessel functions of the first kind with real order and complex arguments.  bessel_K0  Evaluates the real modified Bessel function of the second kind of order zero K <sub>0</sub> (x).  Evaluates the real modified Bessel function of the second kind of order zero K <sub>0</sub> (x).	bessel_Jx	
bessel_Yx  Evaluates a sequence of Bessel functions of the second kind with real order and complex arguments.  bessel_I0  Evaluates the real modified Bessel function of the first kind of order zero Io(x).  bessel_exp_I0  Evaluates the exponentially scaled modified Bessel function of the first kind of order zero.  bessel_I1  Evaluates the real modified Bessel function of the first kind of order one In(x).  bessel_exp_I1  Evaluates the exponentially scaled modified Bessel function of the first kind of order one.  bessel_Ix  Evaluates a sequence of modified Bessel functions of the first kind with real order and complex arguments.  bessel_K0  Evaluates the real modified Bessel function of the second kind of order zero Ko(x).  Evaluates the real modified Bessel function of the second kind of order zero Ko(x).	bessel_Y0	Evaluates the real Bessel function of the second kind of order zero $Y_0(x)$ .
and complex arguments.  Evaluates the real modified Bessel function of the first kind of order zero $I_0(x)$ .  Evaluates the exponentially scaled modified Bessel function of the first kind of order zero.  bessel_l1  Evaluates the real modified Bessel function of the first kind of order one $I_1(x)$ .  Evaluates the real modified Bessel function of the first kind of order one $I_1(x)$ .  Evaluates the exponentially scaled modified Bessel function of the first kind of order one.  Evaluates a sequence of modified Bessel functions of the first kind with real order and complex arguments.  Evaluates the real modified Bessel function of the second kind of order zero $K_0(x)$ .  Evaluates the real modified Bessel function of the second kind of order zero $K_0(x)$ .	bessel_Y1	Evaluates the real Bessel function of the second kind of order one $Y_1(x)$ .
bessel_exp_I0  Evaluates the exponentially scaled modified Bessel function of the first kind of order zero.  bessel_exp_I1  Evaluates the real modified Bessel function of the first kind of order one I <sub>1</sub> (x).  bessel_exp_I1  Evaluates the exponentially scaled modified Bessel function of the first kind of order one.  bessel_lx  Evaluates a sequence of modified Bessel functions of the first kind with real order and complex arguments.  bessel_K0  Evaluates the real modified Bessel function of the second kind of order zero K <sub>0</sub> (x).  Evaluates the exponentially scaled modified Bessel function of the second kind of order zero K <sub>0</sub> (x).	bessel_Yx	
bessel_l1  Evaluates the real modified Bessel function of the first kind of order one $I_1(x)$ .  bessel_exp_l1  Evaluates the exponentially scaled modified Bessel function of the first kind of order one.  bessel_lx  Evaluates a sequence of modified Bessel functions of the first kind with real order and complex arguments.  bessel_K0  Evaluates the real modified Bessel function of the second kind of order zero $K_0(x)$ .  Evaluates the exponentially scaled modified Bessel function of the second kind of order zero $K_0(x)$ .	bessel_I0	Evaluates the real modified Bessel function of the first kind of order zero $I_0(x)$ .
bessel_exp_I1  Evaluates the exponentially scaled modified Bessel function of the first kind of order one.  bessel_lx  Evaluates a sequence of modified Bessel functions of the first kind with real order and complex arguments.  bessel_K0  Evaluates the real modified Bessel function of the second kind of order zero $K_0(x)$ .  Evaluates the real modified Bessel function of the second kind of order zero $K_0(x)$ .	bessel_exp_l0	
bessel_lx  Evaluates a sequence of modified Bessel functions of the first kind with real order and complex arguments.  bessel_K0  Evaluates the real modified Bessel function of the second kind of order zero $K_0(x)$ .  Evaluates the exponentially scaled modified Bessel function of the second kind	bessel_l1	Evaluates the real modified Bessel function of the first kind of order one $I_1(x)$ .
bessel_K0  Evaluates the real modified Bessel function of the second kind of order zero $K_0(x)$ .  Evaluates the exponentially scaled modified Bessel function of the second kind	bessel_exp_l1	
bessel_K0 $K_0(x)$ .  Evaluates the exponentially scaled modified Bessel function of the second kind	bessel_lx	
	bessel_K0	
	bessel_exp_K0	

	BESSEL FUNCTIONS	
	FUNCTION	DESCRIPTION
	bessel_K1	Evaluates the exponentially scaled modified Bessel function of the second kind of order one.
	bessel_exp_K1	Evaluates the exponentially scaled modified Bessel function of the second kind of order one.
_	bessel_Kx	Evaluates a sequence of modified Bessel functions of the second kind with real order and complex arguments.
	ELLIPTIC INTEGRALS	
	FUNCTION	DESCRIPTION
-	FUNCTION	DESCRIPTION
	elliptic_integral_K	Evaluates the complete elliptic integral of the kind $K(x)$ .
	elliptic_integral_E	Evaluates the complete elliptic integral of the second kind $E(x)$ .
	elliptic_integral_RF	Evaluates Carlson's elliptic integral of the first kind $R_F(x, y, z)$ .
	elliptic_integral_RD	Evaluates Carlson's elliptic integral of the second kind $R_D(x, y, z)$ .
	elliptic_integral_RJ	Evaluates Carlson's elliptic integral of the third kind $R_J(x, y, z, \rho)$ .
	elliptic_integral_RC	Evaluates an elementary integral from which inverse circular functions, logarithms, and inverse hyperbolic functions can be computed.
	FRESNEL INTEGRALS	
_	FUNCTION	DESCRIPTION
	fresnel_integral_C	Evaluates the cosine Fresnel integral.
	fresnel_integral_S	Evaluates the sine Fresnel integral.
	AIRY FUNCTIONS	
_	FUNCTION	DESCRIPTION
	airy_Ai	Evaluates the Airy function.
	airy_Bi	Evaluates the Airy function of the second kind.
_		

AIRY FUNCTIONS	
FUNCTION	DESCRIPTION
 airy_Ai_derivative	Evaluates the derivative of the Airy function.
 airy_Bi_derivative	Evaluates the derivative of the Airy function of the second kind.
KELVIN FUNCTIONS	
FUNCTION	DESCRIPTION
 renenen	BESSAII HOW
 kelvin_ber0	Evaluates the Kelvin function of the first kind, ber, of order zero.
 kelvin_bei0	Evaluates the Kelvin function of the first kind, bei, of order zero.
 kelvin_ker0	Evaluates the Kelvin function of the second kind, ker, of order zero.
 kelvin_kei0	Evaluates the Kelvin function of the second kind, kei, of order zero.
 kelvin_ber0_derivative	Evaluates the derivative of the Kelvin function of the first kind, ber, of order zero.
kelvin_bei0_derivative	Evaluates the derivative of the Kelvin function of the first kind, bei, of order zero.
kelvin_ker0_derivative	Evaluates the derivative of the Kelvin function of the second kind, ker, of order zero.
kelvin_kei0_derivative	Evaluates the derivative of the Kelvin function of the second kind, $\mathtt{kei},$ of order zero.
STATISTICAL FUNCTIONS	
 FUNCTION	DESCRIPTION
normal_cdf	Evaluates the standard normal (Gaussian) distribution function.
normal_inverse_cdf	Evaluates the inverse of the standard normal (Gaussian) distribution function.
chi_squared_cdf	Evaluates the chi-squared distribution function.
chi_squared_inverse_cdf	Evaluates the inverse of the chi-squared distribution function.

STATISTICAL FUNCTIONS		
	FUNCTION	DESCRIPTION
	F_cdf	Evaluates the <i>F</i> distribution function.
	F_inverse_cdf	Evaluates the inverse of the <i>F</i> distribution function.
	t_cdf	Evaluates the Student's <i>t</i> distribution function.
	t_inverse_cdf	Evaluates the inverse of the Student's <i>t</i> distribution function.
	gamma_cdf	Evaluates the gamma distribution function.
	binomial_cdf	Evaluates the binomial distribution function.
	hypergeometric_cdf	Evaluates the hypergeometric distribution function.
	poisson_cdf	Evaluates the Poisson distribution function.
	beta_cdf	Evaluates the beta distribution function.
	beta_inverse_cdf	Evaluates the inverse of the beta distribution function.
	bivariate_normal_cdf	Evaluates the bivariate normal distribution function.
	FINANCIAL FUNCTIONS	
	FUNCTION	DESCRIPTION
	cumulative_interest	Evaluates the cumulative interest paid between two periods.
	cumulative_principal	Evaluates the cumulative principal paid between two periods.
	depreciation_db	Evaluates the depreciation of an asset using the fixed-declining balance method.
	depreciation_ddb	Evaluates the depreciation of an asset using the double-declining balance method.

EINANCIAL FUNCTIONS	FINANCIAL FUNCTIONS	
FUNCTION	DESCRIPTION	
depreciation_sIn	Evaluates the depreciation of an asset using the straight-line method.	
depreciation_syd	Evaluates the depreciation of an asset using the sum-of-years digits method.	
depreciation_vdb	Evaluates the depreciation of an asset for any given period using the variable-declining balance method.	
dollar_decimal	Converts a fractional price to a decimal price.	
dollar_fraction	Converts a decimal price to a fractional price.	
effective_rate	Evaluates the effective annual interest rate.	
future_value	Evaluates an investment's future value.	
future_value_schedule	Evaluates the future value of an initial principal taking into consideration a schedule of compound interest rates.	
interest_payment	Evaluates the interest payment for an investment for a given period.	
interest_rate_annuity	Evaluates an annuity's interest rate per period.	
internal_rate_of_return	Evaluates the internal rate of return for a schedule of cash flows.	
internal_rate_schedule	Evaluates the internal rate of return for a schedule of cash flows. It is not necessary that the cash flows be periodic.	
modified_internal_rate	Evaluates the modified internal rate of return for a schedule of periodic cash flows.	
net_present_value	Evaluates an investment's net present value. The calculation is based on a schedule of periodic cash flows and a discount rate.	
nominal_rate	Evaluates the nominal annual interest rate.	
number_of_periods	Evaluates the number of periods for an investment for which periodic and constant payments are made and the interest rate is constant.	

FINANCIAL FUNCTIONS	
FUNCTION	DESCRIPTION
payment	Evaluates the periodic payment for an investment.
present_value	Evaluates the net present value of a stream of equal periodic cash flows, which are subject to a given discount rate.
present_value_schedule	Evaluates the present value for a schedule of cash flows. It is not necessary that the cash flows be periodic.
principal_payment	Evaluates the payment on the principal for a specified period.
POND FUNCTIONS	
BOND FUNCTIONS FUNCTION	DESCRIPTION
accr_interest_maturity	Evaluates the interest that has accrued on a security, which pays interest at maturity.
accr_interest_periodic	Evaluates the interest that has accrued on a security, which pays interest periodically.
bond_equivalent_yield	Evaluates a Treasury bill's bond-equivalent yield.
convexity	Evaluates the convexity for a security.
coupon_days	Evaluates the number of days in the coupon period containing the settlement date.
coupon_number	Evaluates the number of coupons payable between the settlement date and the maturity date.
days_before_settlement	Evaluates the number of days starting with the beginning of the coupon period and ending with the settlement date.
days_to_next_coupon	Evaluates the number of days starting with the settlement date and ending with the next coupon date.
depreciation_amordegrc	Evaluates the depreciation for each accounting period.
depreciation_amorlinc	Evaluates the depreciation for each accounting period. This function is similar to depreciation amordegre, except that depreciation amordegre has a depreciation coefficient that is applied during the evaluation that is based on the asset life.
discount_price	Evaluates a discounted security's price per \$100 face value.

BOND FUNCTIONS	
FUNCTION	DESCRIPTION
discount_rate	Evaluates a security's discount rate.
discount_yield	Evaluates a discounted security's annual yield.
duration	Evaluates a security's annual duration where the security has periodic interest payments.
interest_rate_security	Evaluates a fully invested security's interest rate.
modified_duration	Evaluates a security's modified Macauley duration assuming a par value of \$100.
next_coupon_date	Evaluates the first coupon date that follows the settlement date.
previous_coupon_date	Evaluates the coupon date that immediately precedes the settlement date.
price	Evaluates a security's price per \$100 face value. The security pays periodic interest.
price_maturity	Evaluates a security's price per \$100 face value. The security pays interest at maturity.
received_maturity	Evaluates the amount one receives when a fully invested security reaches the maturity date.
treasury_bill_price	Evaluates a Treasury bill's price per \$100 face value.
treasury_bill_yield	Evaluates a Treasury bill's yield.
year_fraction	Evaluates the fraction of a year represented by the number of whole days between two dates.
yield_maturity	Evaluates a security's annual yield. The security pays interest at maturity.
yield_periodic	Evaluates a security's yield. The security pays periodic interest.

# **CHAPTER 10: STATISTICS AND RANDOM NUMBER GENERATION**

STATISTICS	
FUNCTION	DESCRIPTION
simple_statistics	Computes basic univariate statistics.
table_oneway	Tallies observations into a one-way frequency table.
chi_squared_test	Performs a chi-squared goodness-of-fit test.
covariances	Computes the sample variance-covariance or correlation matrix.
regression	Fits a multiple linear regression model using least-squares.
poly_regression	Performs a polynomial least-squares regression.
ranks	Computes the ranks, normal scores, or exponential scores for a vector of observations.
RANDOM NUMBERS	
FUNCTION	DESCRIPTION
random_seed_get	Retrieves the current value of the seed used in the IMSL random number generators.
random_seed_get random_seed_set	
	generators.
random_seed_set	Initializes a random seed for use in the IMSL random number generators.  Selects the uniform (0,1) multiplicative congruential pseudorandom number
random_seed_set random_option	Initializes a random seed for use in the IMSL random number generators.  Selects the uniform (0,1) multiplicative congruential pseudorandom number generator.
random_seed_set random_option random_uniform	Initializes a random seed for use in the IMSL random number generators.  Selects the uniform (0,1) multiplicative congruential pseudorandom number generator.  Generates pseudorandom numbers from a uniform (0,1) distribution.

RANDOM NUMBERS	
FUNCTION	DESCRIPTION
random_beta	Generates pseudorandom numbers from a beta distribution.
random_exponential	Generates pseudorandom numbers from a standard exponential distribution.
faure_next_point	Computes a shuffled Faure sequence.

# **CHAPTER 11: PRINTING FUNCTIONS**

PRINT	
FUNCTION	DESCRIPTION
write_matrix	Prints a rectangular matrix (or vector) stored in contiguous memory locations.
page	Sets or retrieves the page width or length.
write_options	Sets or retrieves an option for printing a matrix.

# **CHAPTER 12: UTILITIES**

SET OUTPUT FILES	
FUNCTION	DESCRIPTION
output_file	Sets the output file or the error message output file.
version	Returns integer information describing the version of the library, license number, operating system, and compiler.
TIME AND DATE	
FUNCTION	DESCRIPTION
ctime	Returns the number of CPU seconds used.
date_to_days	Computes the number of days from January 1, 1900, to the given date.

TIME AND DATE	
 days_to_date	Gives the date corresponding to the number of days since January 1, 1900.
ERROR HANDLING	
FUNCTION	DESCRIPTION
error_options	Sets various error handling options.
 error_type	Gets the type corresponding to the error message from the last function called.
 error_message	Gets the text of the error message from the last function called.
error_code	Gets the code corresponding to the error message from the last function called.
initialize_error_handler	Initializes the IMSL C Library error handling system.
set_user_fcn_return_flag	Indicates a condition has occurred in a user-supplied function necessitating a return to the calling IMSL C Library function.
C RUNTIME LIBRARY	
FUNCTION	DESCRIPTION
free	Frees memory returned from an IMSL C Math Library function.
fopen	Opens a file using the C runtime library used by the IMSL C Math Library.
fclose	Closes a file opened by imsl_fopen.
OPEN MP	
FUNCTION	DESCRIPTION
omp_options	Sets various OpenMP options.

CONSTANTS	
FUNCTION	DESCRIPTION
constant	Returns the value of various mathematical and physical constants.
machine (float)	Returns information describing the computer's floating-point arithmetic.
 machine (integer)	Returns integer information describing the computer's arithmetic.
 sort	Sorts a vector by algebraic value. Optionally, a vector can be sorted by absolute value, and a sort permutation can be returned.
 sort_integer	Sorts an integer vector by algebraic value. Optionally, a vector can be sorted by absolute value, and a sort permutation can be returned.
COMPUTING VECTOR NORMS	
FUNCTION	DESCRIPTION
vector_norm	Computes various norms of a vector or the difference of two vectors.
vector_norm (complex)	Computes various norms of a vector or the difference of two vectors
LINEAR ALGEBRA SUPPORT	
FUNCTION	DESCRIPTION
mat_mul_rect	Computes the transpose of a matrix, a matrix-vector product, a matrix-matrix product, the bilinear form, or any triple product.
mat_mul_rect (complex)	Computes the transpose of a matrix, the conjugate-transpose of a matrix, a matrix-vector product, a matrix-matrix product, the bilinear form, or any triple product.
 mat_mul_rect_band	Computes the transpose of a matrix, a matrix-vector product, or a matrix-matrix product, all matrices stored in band form.
 mat_mul_rect_band (complex)	Computes the transpose of a matrix, a matrix-vector product, or a matrix-matrix product, all matrices of complex type and stored in band form.
 mat_mul_rect_coordinate	Computes the transpose of a matrix, a matrix-vector product, or a matrix-matrix product, all matrices stored in sparse coordinate form.
 mat_mul_rect_coordinate (complex)	Computes the transpose of a matrix, a matrix-vector product, or a matrix-matrix product, all matrices stored in sparse coordinate form.
mat_add_band	Adds two band matrices, both in band storage mode, $\textit{C} \leftarrow \alpha \textit{A} + \beta \textit{B}$ .

LINEAR ALGEBRA SUPPORT	
mat_add_band (complex)	Adds two band complex matrices, both in band storage mode, $C \leftarrow \alpha A + \beta B$ .
mat_add_coordinate	Performs element-wise addition of two real matrices stored in coordinate format, $C \leftarrow \alpha A + \beta B$ .
 mat_add_coordinate (complex)	Performs element-wise addition on two complex matrices stored in coordinate format, $C \leftarrow \alpha A + \beta B$ .
matrix_norm	Computes various norms of a rectangular matrix.
matrix_norm_band	Computes various norms of a matrix stored in band storage mode.
matrix_norm_coordinate	Computes various norms of a matrix stored in coordinate format.
generate_test_band	Generates test matrices of class $E(n, c)$ . Returns in band or band symmetric format.
generate_test_band (complex)	Generates test matrices of class $E_{\rm c}(n,c)$ . Returns in band or band symmetric format
generate_test_coordinate	Generates test matrices of class $D(n, c)$ and $E(n, c)$ . Returns in either coordinate format.
generate_test_coordinate (complex)	Generates test matrices of class $D(n, c)$ and $E(n, c)$ . Returns in either coordinate or band storage format, where possible.
GPU SUPPORT	
FUNCTION	DESCRIPTION
cuda_get	Gets the threshold used by the specified function to determine if the $NVIDIA^{\otimes}$ $CUDA^{^{\text{TM}}}$ To algorithm will be used.
cuda_set	Sets the threshold used by the specified function to determine if the $\mathit{NVIDIA}^{\otimes}$ $\mathit{CUDA}^{^{TM}}$ $\mathit{To}$ algorithm will be used.
cuda_free	Releases NVIDIA memory allocated by the IMSL C Library.

NUMERIC UTILITIES FUNCTION	DESCRIPTION
c_neg	Changes the sign of a complex number.
c_add	Adds two complex numbers.
c_sub	Subtracts a complex number from a complex number.
c_mul	Multiplies two complex numbers.
c_div	Divides a complex number by a complex number.
c_eq	Tests for equality of two complex numbers.
cz_convert	Truncates a double precision complex number to a single precision complex number.
zc_convert	Increases precision of a single precision complex number to a double precision complex number.
cf_convert	Makes a complex number from an ordered pair.
c_conjg	Conjugates a complex number.
c_abs	Computes a magnitude of a complex number.
c_arg	Computes an angle of a complex number.
c_sqrt	Computes a square root of a complex number.
c_cos	Computes a trigonometric cosine of a complex number.
c_sin	Computes a trigonometric sine of a complex number.
c_exp	Computes an exponential function of a complex number.

NUMERIC UTILITIES	NUMERIC UTILITIES	
c_log	Computes a natural logarithm of a complex number.	
cf_power	Computes a complex number raised to a real power.	
cc_power	Computes a complex number raised to a complex power.	
fi_power	Computes a real number raised to an integral power.	
ii_power	Computes an integer raised to an integral power.	

# **IMSL C STAT LIBRARY**

### **CHAPTER 1: BASIC STATISTICS**

SIMPLE SUMMARY STATISTICS	
FUNCTION	DESCRIPTION
simple_statistics	Computes basic univariate statistics.
empirical_quantiles	Computes empirical quantiles.
normal_one_sample	Computes statistics for mean and variance inferences using a sample from a normal population.
normal_two_sample	Computes statistics for mean and variance inferences using samples from two normal populations.
TABULATE, SORT, AND RANK	
FUNCTION	DESCRIPTION
table_oneway	Tallies observations into a one-way frequency table.
table_twoway	Tallies observations into a two-way frequency table.
sort_data	Sorts observations by specified keys, with option to tally cases into a multi-way frequency table.
ranks	Computes the ranks, normal scores, or exponential scores for a vector of observations.
	simple_statistics  empirical_quantiles  normal_one_sample  normal_two_sample  TABULATE, SORT, AND RANK FUNCTION  table_oneway  table_twoway  sort_data

# **CHAPTER 2: REGRESSION**

MULTIVARIATE LINEAR REGRESSION — MODEL FITTING	
FUNCTION	DESCRIPTION
regressors_for_glm	Generates regressors for a general linear model.
regression	Fits a multiple linear regression model using least squares.

	MULTIVARIATE LINEAR REGRESSION — STATISTICAL INFERENCE AND DIAGNOSTICS		
	FUNCTION	DESCRIPTION	
	regression_summary	Produces summary statistics for a regression model given the information from the fit.	
	regression_prediction	Computes predicted values, confidence intervals, and diagnostics after fitting a regression model.	
	hypothesis_partial	Constructs a completely testable hypothesis.	
	hypothesis_scph	Sums of cross products for a multivariate hypothesis.	
	hypothesis_test	Tests for the multivariate linear hypothesis.	
	VARIABLE SELECTION		
	FUNCTION	DESCRIPTION	
	regression_selection	Selects the best multiple linear regression models.	
	regression_stepwise	Builds multiple linear regression models using forward selection, backward selection, or stepwise selection.	
	POLYNOMIAL AND NONLINEAR REGRESSION		
	FUNCTION	DESCRIPTION	
	poly_regression	Performs a polynomial least-squares regression.	
	poly_prediction	Computes predicted values, confidence intervals, and diagnostics after fitting a polynomial regression model.	
	nonlinear_regression	Fits a nonlinear regression model.	
	nonlinear_optimization	Fits a nonlinear regression model using Powell's algorithm.	

ALTERNATIVES TO LEAST-SQUARES	ALTERNATIVES TO LEAST-SQUARES	
FUNCTION	DESCRIPTION	
Lnorm_regression	Fits a multiple linear regression model using $L_p$ criteria other than least squares.	
pls_regression	Performs partial least-squares (PLS) regression for one or more response variables and one or more predictor variables.	

## **CHAPTER 3: CORRELATION AND COVARIANCE**

VARIANCES, COVARIANCE	VARIANCES, COVARIANCES, AND CORRELATIONS	
FUNCTION	DESCRIPTION	
covariances	Computes the variance-covariance or correlation matrix.	
partial_covariances	Computes a pooled variance-covariance matrix from the observations.	
pooled_covariances	Computes partial correlations or covariances from the covariance or correlation matrix.	
robust_covariances	Computes a robust estimate of a covariance matrix and mean vector.	

### **CHAPTER 4: ANALYSIS OF VARIANCE AND DESIGNED EXPERIMENTS**

GENERAL ANALYSIS	GENERAL ANALYSIS	
FUNCTION	DESCRIPTION	
anova_oneway	Analyzes a one-way classification model.	
ancovar	Analyzes a one-way classification model with covariates.	
anova_factorial	Analyzes a balanced factorial design with fixed effects.	
anova_nested	Analyzes a completely nested random effects model with possibly unequal numbers in the subgroups.	
anova_balanced	Analyzes a balanced complete experimental design for a fixed, random, or mixed model.	

DESIGNED EXPERIMENTS	
FUNCTION	DESCRIPTION
crd_factorial	Analyzes data from balanced and unbalanced completely randomized experiments.
rcbd_factorial	Analyzes data from balanced and unbalanced randomized complete-block experiments.
latin_square	Analyzes data from latin-square experiments.
lattice	Analyzes balanced and partially-balanced lattice experiments.
split_plot	Analyzes a wide variety of split-plot experiments with fixed, mixed, or random factors.
split_split_plot	Analyzes data from split-split-plot experiments.
strip_plot	Analyzes data from strip-plot experiments.
strip_split_plot	Analyzes data from strip-split-plot experiments.
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UTILITIES	
 FUNCTION	DESCRIPTION
homogeneity	Conducts Bartlett's and Levene's tests of the homogeneity of variance assumption in analysis of variance.
multiple_comparisons	Compares differences among averages using the SNK, LSD, Tukey's, Duncan's, and Bonferroni's multiple comparisons tests.
false_discovery_rates	Calculates the False Discovery Rate (FDR) <i>q</i> -values corresponding to a set of <i>p</i> -values resulting from multiple simultaneous hypothesis tests.
yates	Estimates missing observations in designed experiments using Yate's method.

#### **CHAPTER 5: CATEGORICAL AND DISCRETE DATA ANALYSIS**

STATISTICS IN THE TWO-WAY CON	STATISTICS IN THE TWO-WAY CONTINGENCY TABLE	
FUNCTION	DESCRIPTION	
contingency_table	Performs a chi-squared analysis of a two-way contingency table.	
exact_enumeration	Computes exact probabilities in a two-way contingency table using the total enumeration method.	
exact_network	Computes exact probabilities in a two-way contingency table using the network algorithm.	
CATEGORICAL MODELS		
FUNCTION	DESCRIPTION	
categorical_glm	Analyzes categorical data using logistic, Probit, Poisson, and other generalized linear models.	
logistic_regression	Fits a binomial or multinomial logistic regression model using iteratively reweighted least-squares.	
logistic_reg_predict	Predicts a binomial or multinomial outcome given an estimated model and new values of the independent variables.	

### **CHAPTER 6: NONPARAMETRIC STATISTICS**

ONE SAMPLE TESTS — NONPARAMETRI	ONE SAMPLE TESTS — NONPARAMETRIC STATISTICS	
FUNCTION	DESCRIPTION	
sign_test	Performs a sign test.	
wilcoxon_sign_rank	Performs a Wilcoxon signed rank test.	
noether_cyclical_trend	Performs the Noether's test for cyclical trend.	
cox_stuart_trends_test	Performs the Cox and Stuart sign test for trends in location and dispersion.	
tie_statistics	Computes tie statistics for a sample of observations.	

TWO OR MORE SAMPLES	TWO OR MORE SAMPLES	
FUNCTION	DESCRIPTION	
wilcoxon_rank_sum	Performs a Wilcoxon rank sign test.	
kruskal_wallis_test	Performs a Kruskal-Wallis test for identical population medians.	
friedmans_test	Performs Friedman's test for a randomized complete block design.	
cochran_q_test	Performs Cochran's Q test for related observations.	
k_trends_test	Performs <i>k</i> -sample trends test against ordered alternatives.	

# **CHAPTER 7: TESTS OF GOODNESS-OF-FIT**

GENERAL GOODNESS-OF-FIT TESTS FO	GENERAL GOODNESS-OF-FIT TESTS FOR A SPECIFIED DISTRIBUTION	
FUNCTION	DESCRIPTION	
chi_squared_test	Performs a chi-squared goodness-of-fit test.	
shapiro_wilk_normality_test	Performs the Shapiro-Wilk test for normality.	
lilliefors_normality_test	Performs a Lilliefors test for normality.	
chi_squared_normality_test	Performs a chi-squared test for normality.	
kolmogorov_one	Performs a Kolmogorov-Smirnov's one-sample test for continuous distributions.	
kolmogorov_two	Performs a Kolmogorov-Smirnov's two-sample test.	
multivar_normality_test	Computes Mardia's multivariate measures of skewness and kurtosis and tests for multivariate normality.	
ad_normality_test	Performs an Anderson-Darling test for normality.	
cvm_normality_test	Performs a Cramer-von Mises test for normality.	

TESTS FOR RANDOMNESS	
FUNCTION	DESCRIPTION
randomness_test	Performs a test for randomness.

# **CHAPTER 8: TIME SERIES ANALYSIS AND FORECASTING**

ARIMA MODELS	
FUNCTION	DESCRIPTION
arma	Computes least-square estimates of parameters for an ARMA (autoregressive, moving average) model.
max_arma	Exact maximum likelihood estimation of the parameters in a univariate ARMA time series model.
arma_forecast	Computes forecasts and their associated probability limits for an ARMA model.
regression_arima	Fits a univariate ARIMA ( $p$ , $d$ , $q$ ) time series model with the inclusion of one or more regression variables.

### **AUTOMATIC ARIMA SELECTION AND FITTING UTILITIES**

FUNCTION	DESCRIPTION
auto_uni_ar	Automatic selection and fitting of a univariate autoregressive time series model. The lag for the model is automatically selected using Akaike's information criterion (AIC).
seasonal_fit	Estimates the optimum seasonality parameters for a time series using an autoregressive model, $AR(p)$ , to represent the time series.
ts_outlier_identification	Detects and determines outliers and simultaneously estimates the model parameters in a time series whose underlying outlier-free series follows a general seasonal or nonseasonal ARMA model.
ts_outlier_forecast	Computes forecasts, their associated probability limits and weights for an outlier contaminated time series whose underlying outlier free series follows a general seasonal or nonseasonal ARMA model.
auto_arima	Automatically identifies time series outliers, determines parameters of a multiplicative seasonal ARIMA $(p,0,q)\times(0,d,0)_S$ model and produces forecasts that incorporate the effects of outliers whose effects persist beyond the end of the series.
auto_parm	Estimates structural breaks in non-stationary univariate time series.

	BAYESIAN TIME SERIES ESTIMATION	
	FUNCTION	DESCRIPTION
	bayesian_seasonal_adj	Decomposes a time series into trend, seasonal, and an error component.
	MODEL CONSTRUCTION AND EVALUATION	ON UTILITIES
	FUNCTION	DESCRIPTION
	box_cox_transform	Performs a Box-Cox transformation.
	difference	Differences a seasonal or nonseasonal time series.
	autocorrelation	Computes the sample autocorrelation function of a stationary time series.
	crosscorrelation	Computes the sample cross-correlation function of two stationary time series.
	multi_crosscorrelation	Computes the multichannel cross-correlation function of two mutually stationary multichannel time series.
	partial_autocorrelation	Computes the sample partial autocorrelation function of a stationary time series
	lack_of_fit	Performs lack-of-fit test for an univariate time series or transfer function given the appropriate correlation function.
	estimate_missing	Estimates missing values in a time series.
	EXPONENTIAL SMOOTHING METHODS	
	FUNCTION	DESCRIPTION
	hw_time_series	Calculates parameters and forecasts using the Holt-Winters Multiplicative or Additive forecasting method for seasonal data.
GARCH MODELING		
	FUNCTION FUNCTION	DESCRIPTION
	garch	Computes estimates of the parameters of a $GARCH(p,q)$ model.

STATE-SPACE MODELS	
FUNCTION	DESCRIPTION
kalman	Performs Kalman filtering and evaluates the likelihood function for the state-space model.
AUTO-REGRESSION AND ERROR CORRECTION	
FUNCTION	DESCRIPTION
vector_autoregression	Estimates a vector auto-regressive time series model with optional moving average components.
	Kalman  AUTO-REGRESSION AND ERROR CORRE

# **CHAPTER 9: MULTIVARIATE ANALYSIS**

HIERARCHICAL CLUSTER ANALYSIS	
FUNCTION	DESCRIPTION
dissimilarities	Computes a matrix of dissimilarities (or similarities) between the columns (or rows) of a matrix.
cluster_hierarchical	Performs a hierarchical cluster analysis given a distance matrix.
 cluster_number	Computes cluster membership for a hierarchical cluster tree.
K-MEANS CLUSTER ANALYSIS	
FUNCTION	DESCRIPTION
cluster_k_means	Performs a <i>K</i> -means (centroid) cluster analysis.
PRINCIPAL COMPONENT ANALYSIS	
FUNCTION	DESCRIPTION
principal_components	Computes principal components.
FACTOR ANALYSIS	
 FUNCTION	DESCRIPTION
factor_analysis	Extracts initial factor-loading estimates in factor analysis with rotation options.
discriminant_analysis	Performs discriminant function analysis.

#### **CHAPTER 10: SURVIVAL AND RELIABILITY ANALYSIS**

SURVIVAL ANALYSIS	
FUNCTION	DESCRIPTION
kaplan_meier_estimates	Computes Kaplan-Meier estimates of survival probabilities in stratified samples.
prop_hazards_gen_lin	Analyzes survival and reliability data using Cox's proportional hazards model.
survival_glm	Analyzes censored survival data using a generalized linear model.
survival_estimates	Estimates survival probabilities and hazard rates for the various parametric models.
RELIABILITY ANALYSIS	
FUNCTION	DESCRIPTION
nonparam_hazard_rate	Estimates a reliability hazard function using a nonparametric approach.
ACTUARIAL TABLES	
FUNCTION	DESCRIPTION
life_tables	Produces population and cohort life tables.

## **CHAPTER 11: PROBABILITY DISTRIBUTION FUNCTIONS AND INVERSES**

DISCRETE RANDOM VARIABLES	
FUNCTION	DESCRIPTION
binomial_cdf	Evaluates the binomial distribution function.
binomial_pdf	Evaluates the binomial probability function.
geometric_cdf	Evaluates the discrete geometric cumulative distribution function.
geometric_inverse_cdf	Evaluates the inverse of the discrete geometric cumulative distribution function.

DISCRETE RANDOM VARIABLES FUNCTION	DESCRIPTION
geometric_pdf	Evaluates the discrete geometric probability density function.
 hypergeometric_cdf	Evaluates the hypergeometric distribution function.
 hypergeometric_pdf	Evaluates the hypergeometric probability function.
 poisson_cdf	Evaluates the Poisson distribution function.
 poisson_pdf	Evaluates the Poisson probability function.
discrete_uniform_cdf	Evaluates the discrete uniform cumulative distribution function.
 discrete_uniform_inverse_cdf	Evaluates the inverse of the discrete uniform cumulative distribution function.
 discrete_uniform_pdf	Evaluates the discrete uniform probability density function .
CONTINUOUS RANDOM VARIABLES	
FUNCTION	DESCRIPTION
beta_cdf	Evaluates the beta probability distribution function.
beta_inverse_cdf	Evaluates the inverse of the beta distribution function.
non_central_beta_cdf	Evaluates the noncentral beta cumulative distribution function.
non_central_beta_inverse_cdf	Evaluates the inverse of the noncentral beta cumulative distribution function.
non_central_beta_pdf	Evaluates the noncentral beta probability density function.
bivariate_normal_cdf	Evaluates the bivariate normal distribution function.
chi_squared_cdf	Evaluates the chi-squared distribution function.

CONTINUOUS RANDOM VARIABLES	
FUNCTION	DESCRIPTION
chi_squared_inverse_cdf	Evaluates the inverse of the chi-squared distribution function.
complementary_chi_squared_cdf	Evaluates the complement of the chi-squared distribution.
complementary_F_cdf	Evaluates the complement of the $F$ distribution function.
complementary_t_cdf	Evaluates the complement of the Student's $t$ distribution function.
exponential_cdf	Evaluates the exponential cumulative distribution function.
exponential_inverse_cdf	Evaluates the inverse of the exponential cumulative distribution function.
exponential_pdf	Evaluates the exponential probability density function.
F_cdf	Evaluates the $F$ distribution function.
F_inverse_cdf	Evaluates the inverse of the <i>F</i> distribution function.
gamma_cdf	Evaluates the gamma distribution function.
gamma_inverse_cdf	Evaluates the inverse of the gamma distribution function.
lognormal_cdf	Evaluates the lognormal cumulative distribution function.
lognormal_inverse_cdf	Evaluates the inverse of the lognormal cumulative distribution function.
lognormal_pdf	Evaluates the lognormal probability density function.
multivariate_normal_cdf	Evaluates the cumulative distribution function for the multivariate normal distribution.
non_central_chi_sq	Evaluates the noncentral chi-squared distribution function.

CONTINUOUS RANDOM VARIABLES	
FUNCTION	DESCRIPTION
non_central_chi_sq_inv	Evaluates the inverse of the noncentral chi-squared function.
non_central_chi_sq_pdf	Evaluates the noncentral chi-squared probability density function.
non_central_F_cdf	Evaluates the noncentral $F$ cumulative distribution function.
complementary_non_central_F_cdf	Evaluates the complementary noncentral $F$ cumulative distribution function.
non_central_F_inverse_cdf	Evaluates the inverse of the noncentral $F$ cumulative distribution function.
non_central_F_pdf	Evaluates the noncentral $F$ probability density function.
non_central_t_cdf	Evaluates the noncentral Student's <i>t</i> distribution function.
non_central_t_inv_cdf	Evaluates the inverse of the noncentral Student's <i>t</i> distribution function.
non_central_t_pdf	Evaluates the noncentral Student's <i>t</i> probability density function.
pareto_cdf	Evaluates the Pareto cumulative probability distribution function.
pareto_pdf	Evaluates the Pareto probability density function.
normal_cdf	Evaluates the standard normal (Gaussian) distribution function.
normal_inverse_cdf	Evaluates the inverse of the standard normal (Gaussian) distribution function.
t_cdf	Evaluates the Student's <i>t</i> distribution function.
t_inverse_cdf	Evaluates the inverse of the Student's <i>t</i> distribution function.

PARAMETER ESTIMATION	
FUNCTION	DESCRIPTION
max_likelihood_estimates	Calculates maximum likelihood estimates (MLE) for the parameters of one of several univariate probability distributions.

## **CHAPTER 12: RANDOM NUMBER GENERATION**

UNIVARIATE DISCRETE DISTRIBUTIONS	
FUNCTION	DESCRIPTION
random_binomial	Generates pseudorandom binomial numbers from a binomial distribution.
random_geometric	Generates pseudorandom numbers from a geometric distribution.
random_hypergeometric	Generates pseudorandom numbers from a hypergeometric distribution.
random_logarithmic	Generates pseudorandom numbers from a logarithmic distribution.
random_neg_binomial	Generates pseudorandom numbers from a negative binomial distribution.
random_poisson	Generates pseudorandom numbers from a Poisson distribution.
random_uniform_discrete	Generates pseudorandom numbers from a discrete uniform distribution.
random_general_discrete	Generates pseudorandom numbers from a general discrete distribution using an alias method or, optionally, a table lookup method.
discrete_table_setup	Sets up a table to generate pseudorandom numbers from a general discrete distribution.
LININ/ADIATE OCNERNICO DIOTEIRI	
UNIVARIATE CONTINUOUS DISTRIBUTIONS	
FUNCTION	DESCRIPTION
random_beta	Generates pseudorandom numbers from a beta distribution.
random_cauchy	Generates pseudorandom numbers from a Cauchy distribution.

UNIVARIATE CONTINUOUS DISTRIBUTIO	UNIVARIATE CONTINUOUS DISTRIBUTIONS	
FUNCTION	DESCRIPTION	
random_chi_squared	Generates pseudorandom numbers from a chi-squared distribution.	
random_exponential	Generates pseudorandom numbers from a standard exponential distribution.	
random_exponential_mix	Generates pseudorandom mixed numbers from a standard exponential distribution.	
random_gamma	Generates pseudorandom numbers from a standard gamma distribution.	
random_lognormal	Generates pseudorandom numbers from a lognormal distribution.	
random_normal	Generates pseudorandom numbers from a standard normal distribution.	
random_stable	Generates pseudorandom numbers from a stable distribution.	
random_student_t	Generates pseudorandom numbers from a Student's <i>t</i> distribution.	
random_triangular	Generates pseudorandom numbers from a triangular distribution.	
random_uniform	Generates pseudorandom numbers from a uniform (0, 1) distribution.	
random_von_mises	Generates pseudorandom numbers from a von Mises distribution.	
random_weibull	Generates pseudorandom numbers from a Weibull distribution.	
random_general_continuous	Generates pseudorandom numbers from a general continuous distribution.	
continuous_table_setup	Sets up a table to generate pseudorandom numbers from a general continuous distribution.	

MULTIVARIATE CONTINUOUS DISTRIBUT	MULTIVARIATE CONTINUOUS DISTRIBUTIONS	
FUNCTION	DESCRIPTION	
random_normal_multivariate	Generates pseudorandom numbers from a multivariate normal distribution.	
random_orthogonal_matrix	Generates a pseudorandom orthogonal matrix or a correlation matrix.	
random_mvar_from_data	Generates pseudorandom numbers from a multivariate distribution determined from a given sample.	
random_multinomial	Generates pseudorandom numbers from a multinomial distribution.	
random_sphere	Generates pseudorandom points on a unit circle or K-dimensional sphere.	
random_table_twoway	Generates a pseudorandom two-way table.	
random_mvar_gaussian_copula	Given a Cholesky factorization of a correlation matrix, generates pseudorandom numbers from a Gaussian Copula distribution.	
random_mvar_t_copula	Given a Cholesky factorization of a correlation matrix, generates pseudorandom numbers from a Student's <i>t</i> Copula distribution.	
canonical_correlation	Given an input array of deviate values, generates a canonical correlation array.	
ORDER STATISTICS		
FUNCTION	DESCRIPTION	
random_order_normal	Generates pseudorandom order statistics from a standard normal distribution.	
random_order_uniform	Generates pseudorandom order statistics from a uniform (0, 1) distribution.	
STOCHASTIC PROCESSES		
FUNCTION	DESCRIPTION	
random_arma	Generates a time series from a specific ARMA model.	
random_npp	Generates pseudorandom numbers from a nonhomogeneous Poisson process.	

SAMPLES AND PERMUTATIONS FUNCTION	DESCRIPTION
random_permutation	Generates a pseudorandom permutation.
random_sample_indices	Generates a simple pseudorandom sample of indices.
random_sample	Generates a simple pseudorandom sample from a finite population.
UTILITY FUNCTIONS	
FUNCTION	DESCRIPTION
random_option	Selects the uniform (0, 1) multiplicative congruential pseudorandom number generator.
random_option_get	Retrieves the uniform (0, 1) multiplicative congruential pseudorandom number generator.
random_seed_get	Retrieves the current value of the seed used in the IMSL random number generators.
random_substream_seed_get	Retrieves a seed for the congruential generators that do not do shuffling that will generate random numbers beginning 100,000 numbers farther along.
random_seed_set	Initializes a random seed for use in the IMSL random number generators.
random_table_set	Sets the current table used in the shuffled generator.
random_table_get	Retrieves the current table used in the shuffled generator.
random_GFSR_table_set	Sets the current table used in the GFSR generator.
random_GFSR_table_get	Retrieves the current table used in the GFSR generator.
random_MT32_init	Initializes the 32-bit Mersenne Twister generator using an array.
random_MT32_table_get	Retrieves the current table used in the 32-bit Mersenne Twister generator.
random_MT32_table_set	Sets the current table used in the 32-bit Mersenne Twister generator.

UTILITY FUNCTIONS	
FUNCTION	DESCRIPTION
random_MT64_init	Initializes the 64-bit Mersenne Twister generator using an array.
random_MT64_table_get	Retrieves the current table used in the 64-bit Mersenne Twister generator.
random_MT64_table_set	Sets the current table used in the 64-bit Mersenne Twister generator.
LOW-DISCREPANCY SEQUENCE	
FUNCTION	DESCRIPTION
faure_next_point	Computes a shuffled Faure sequence.

# **CHAPTER 13: DATA MINING**

APRIORI – MARKET BASKET ANALYSIS	APRIORI – MARKET BASKET ANALYSIS	
FUNCTION	DESCRIPTION	
apriori	Computes the frequent itemsets in a transaction set.	
aggr_apriori	Computes the frequent itemsets in a transaction set using aggregation.	
write_apriori_itemsets	Prints frequent itemsets.	
write_association_rules	Prints association rules.	
free_apriori_itemsets	Frees the memory allocated within a frequent itemsets structure.	
free_association_rules	Frees the memory allocated within an association rules structure.	

DECISION TREE	
FUNCTION	DESCRIPTION
decision_tree	Generates a decision tree for a single response variable and two or more predictor variables.
decision_tree_predict	Computes predicted values using a decision tree.
decision_tree_print	Prints a decision tree.
decision_tree_free	Frees the memory associated with a decision tree.

GENETIC ALGORITHM DATA STRUCTURES		
FUNCTION	DESCRIPTION	
ga_chromosome	Creates an <i>Imsls_f_chromosome</i> data structure containing unencoded and encoded phenotype information.	
ga_copy_chromosome	Copies the contents of one chromosome into another chromosome.	
ga_clone_chromosome	Clones an existing chromosome.	
ga_individual	Creates an Imsls_f_individual data structure from user supplied phenotypes.	
ga_copy_individual	Copies the contents of one individual into another individual.	
ga_clone_individual	Clones an existing individual.	
ga_mutate	Performs the mutation operation on an individual's chromosome.	
ga_decode	Decodes an individual's chromosome into its binary, nominal, integer, and real phenotypes.	
ga_encode	Encodes an individual's binary, nominal, integer, and real phenotypes into its chromosome.	
ga_free_individual	Frees memory allocated to an existing individual.	

GENETIC ALGORITHM DATA STRUCTURES	
FUNCTION	DESCRIPTION
ga_population	Creates an Imsls_f_population data structure from user supplied individuals.
ga_random_population	Creates an <i>Imsls_f_population</i> data structure from randomly generated individuals.
ga_copy_population	Copies the contents of one population into another population.
ga_clone_population	Clones an existing population.
ga_grow_population	Adds individuals to an existing population.
ga_merge_population	Creates a new population by merging two populations with identical chromosome structures.
ga_free_population	Frees memory allocated to an existing population.
genetic_algorithm	Optimizes a user defined fitness function using a tailored genetic algorithm.
NAIVE BAYES	
-	
FUNCTION	DESCRIPTION
naive_bayes_trainer	Trains a Naïve Bayes classifier.
naive_bayes_classification	Classifies unknown patterns using a previously trained Naïve Bayes classifier.
nb_classifier_free	Frees memory allocated to an <i>Imsls_f_nb_classifier</i> data structure.
nb_classifier_write	Writes a Naïve Bayes Classifier to an ASCII file for later retrieval using imsls_f_nb_classifier_read.
nb_classifier_read	Retrieves a Naïve Bayes Classifier from a file previously saved using imsls_f_nb_classifier_write.
NEURAL NETWORK DATA STRUCTURES	
FUNCTION	DESCRIPTION
mlff_network_init	Initializes a data structure for training a neural network.

NEURAL NETWORK DATA STRUCTURES	
FUNCTION	DESCRIPTION
mlff_network	Multilayered feedforward neural network.
mlff_network_free	Frees memory allocated for an <i>Imsls_f_NN_Network</i> data structure.
mlff_network_write	Writes a trained neural network to an ASCII file.
mlff_network_read	Retrieves a neural network from a file previously saved.
mlff_initialize_weights	Initializes weights for multilayered feedforward neural networks prior to network training using one of four user selected methods.
FORECASTING NEURAL NETWORKS	
 FUNCTION	DESCRIPTION
mlff_network_trainer	Trains a multilayered feedforward neural network.
 mlff_network_forecast	Calculates forecasts for trained multilayered feedforward neural networks.
CLASSIFICATION NEURAL NETWORKS	
FUNCTION	DESCRIPTION
mlff_classification_trainer	Trains a multilayered feedforward neural network for classification.
mlff_pattern_classification	Calculates classifications for trained multilayered feedforward neural networks.
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PREPROCESSING FILTERS	
 FUNCTION	DESCRIPTION
 scale_filter	Scales or unscales continuous data prior to its use in neural network training, testing, or forecasting.
 time_series_filter	Converts time series data to the format required for processing by a neural network.
time_series_class_filter	Converts time series data sorted within nominal classes in decreasing chronological order to a useful format for processing by a neural network.

	PREPROCESSING FILTERS	PREPROCESSING FILTERS	
	FUNCTION	DESCRIPTION	
	unsupervised_nominal_filter	Converts nominal data into a series of binary encoded columns for input to a neural network. Optionally, it can also reverse the binary encoding, accepting a series of binary encoded columns and returning a single column of nominal classes.	
	unsupervised_ordinal_filter	Converts ordinal data into proportions. Optionally, it can also reverse encoding, accepting proportions and converting them into ordinal values.	
	SELF ORGANIZING MAPS		
	FUNCTION	DESCRIPTION	
	kohonenSOM_trainer	Trains a Kohonen network.	
	kohonenSOM_forecast	Calculates forecasts using a trained Kohonen network.	
	SUPPORT VECTOR MACHINES		
	FUNCTION	DESCRIPTION	
	support_vector_trainer	Trains a Support Vector Machines (SVM) classifier.	
	support_vector_classification	Classifies unknown patterns using a previously trained Support Vector Machines (SVM) model computed by support_vector_trainer.	
	svm_classifier_free	Frees memory allocated to an <code>Imsls_f_svm_model/Imsls_d_svm_model</code> data structure.	

# **CHAPTER 14: PRINTING FUNCTIONS**

	PRINT	
	FUNCTION	DESCRIPTION
	write_matrix	Prints a rectangular matrix (or vector) stored in contiguous memory locations.
	SET	
-	FUNCTION	DESCRIPTION
	page	Sets or retrieves the page width or length.
	write_options	Sets or retrieves an option for printing a matrix.

## **CHAPTER 15: UTILITIES**

SET OUPUT FILES	
FUNCTION	DESCRIPTION
output_file	Sets the output file or the error message output file.
version	Returns integer information describing the version of the library, license number, operating system, and compiler.
ERROR HANDLING	
FUNCTION	DESCRIPTION
error_options	Sets various error handling options.
error_type	Gets the type corresponding to the error message from the last function called.
error_message	Gets the text of the error message from the last function called.
error_code	Returns the code corresponding to the error message from the last function called.
initialize_error_handler	Initializes the IMSL C Stat Library error handling system.
set_user_fcn_return_flag	Indicates a condition has occurred in a user-supplied function necessitating a return to the calling function.

C RUNTIME LIBRARY	
FUNCTION	DESCRIPTION
free	Frees memory returned from an IMSL C Stat Library function.
fopen	Opens a file using the C runtime library used by the IMSL C Stat Library.
fclose	Closes a file opened by imsls_fopen.
ascii_read	Reads freely-formatted ASCII files.
OPENMP	
FUNCTION	DESCRIPTION
omp_options	Sets various OpenMP options.
CONSTANTS	
FUNCTION	DESCRIPTION
machine (integer)	Returns integer information describing the computer's arithmetic.
machine (float)	Returns information describing the computer's floating-point arithmetic.
data_sets	Retrieves a commonly analyzed data set.
MATHEMATICAL SUPPORT	
FUNCTION	DESCRIPTION
mat_mul_rect	Computes the transpose of a matrix, a matrix-vector product, a matrix-matrix product, a bilinear form, or any triple product.
permute_vector	Rearranges the elements of a vector as specified by a permutation.
permute_matrix	Permutes the rows or columns of a matrix.
impute_missing	Locates and optionally replaces dependent variable missing values with nearest neighbor estimates.

MATHEMATICAL SUPPORT	
FUNCTION	DESCRIPTION
binomial_coefficient	Evaluates the binomial coefficient.
 beta	Evaluates the real regularized incomplete beta function.
beta_incomplete	Evaluates the real regularized incomplete beta function.
log_beta	Evaluates the logarithm of the real beta function $\beta(x, y)$ .
gamma	Evaluates the real gamma function.
gamma_incomplete	Evaluates the incomplete gamma function $\gamma(a, x)$ .
log_gamma	Evaluates the logarithm of the absolute value of the gamma function log $ \Gamma(x) $ .
ctime	Returns the number of CPU seconds used.
GPU SUPPORT	
FUNCTION	DESCRIPTION
cuda_get	Gets the threshold used by the specified function to determine if the <i>NVIDIA</i> ® <i>CUDA</i> ™ <i>Toolkit</i> algorithm will be used.
cuda_set	Sets the threshold used by the specified function to determine if the $NVIDIA^{\oplus}\ CUDA^{\top}\ Toolkit$ algorithm will be used.
cuda_free	Releases NVIDIA memory allocated by the IMSL C Library.